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THESIS

ERROR CONTROL IN MODEL FOLLOWING CONTROL SYSTEMS USING CONSTANT ERROR MODEL FOLLOWING

by

Wayne C. Durham

March 1984

Thesis Advisor: Marle D. Hewett

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Error Control in Model Following Control Systems Using Constant Error Model Following

by

Wayne C. Durham
Commander, United States Navy
B.S., U. S. Naval Academy, 1965

Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING
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ABSTRACT

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The most special recognition goes to my wife Kathy who, besides having to put up with a fighter pilot as a husband, had to suffer my obsession with hammering out this theory. She did this with love (always) and patience (usually). This work is dedicated to her with my love.

I. INTRODUCTION

The theories presented in this thesis were developed in the course of researching control techniques for use in aircraft departure prevention. The aim was to apply model following control methods to the nonlinear, time varying and coupled dynamic equations of motion of an aircraft at critical combinations of angle of attack and angular rates. When it was seen that a new development in model following control had evolved from the research, it became the sole subject of the thesis.

Model following control is concerned with techniques which cause a physical plant to behave as much like a model as possible. Motyka [Ref. 1] accomplishes this by solving the plant state equations for their controls and then substituting expressions for the model states and state rates into these equations. That is, assuming that the plant is defined by the linear, small perturbation constant coefficient differential equation:

$$\dot{x}_p = f_1 x_p + g_1 u_p \tag{1}$$

where $\bar{x}_p$ is the state of the plant

u_p is the control

and that a model is given by the corresponding differential equation:

$$\dot{x}_m = f_2 x_m + g_2 u_m \tag{2}$$

It is desired that $x_m = x_p$ and $\dot{x}_m = \dot{x}_p$. The plant control which achieves this can be determined by substituting the desired relationships into the plant equation:

$$\dot{x}_m = f_1 x_m + g_1 u_p \tag{3}$$

And solving for u_p . The result is an expression for the plant inputs which make the plant state equal to that of the model.

Problems with this method arise when equations (1) and (2) are vector equations with fewer controls than states to be controlled. Also, if equation (1), which is a mathematical description of a real, physical plant, fails to describe that plant accurately, errors may be introduced into the response of the system. Finally, errors will occur if the plant and the model do not have the same initial conditions, or equivalently, in the presence of external disturbances.

The first two of these three problems are treated by Moytka [Ref. 1] and Rynaski [Refs. 2 and 3]. They are beyond the scope of this paper. The theories presented

herein apply specifically to the problem of reducing the system error in the third case. The method, called constant error model following control, is developed and demonstrated for nonlinear, time varying, first order, single input-single output systems with additive (linear) control.

II. PROBLEM DESCRIPTION

For the purposes of this discussion, it is assumed that a plant (physical process) is given, and that it is completely described by a first order differential equation with additive control of the form

$$\dot{x}_p = f_1 (x_p , t) + g_1 u_p \tag{1}$$

where x_p is the state of the plant
t is the variable time
u_p is the control input to the plant
f_1 is (in general) time varying and nonlinear in x_p
g_1 is constant

It is desired that the response of the plant be modified in some way. This modified response is completely described by the mathematical model

$$\dot{x}_m = f_2 (x_m , t) + g_2 u_s \tag{2}$$

where x_m is the state of the model which has the desired response
u_s is the control input to the system which incorporates the model and the plant, and
f_2 is (in general) time varying and nonlinear in x_m , and $f_2 \neq f_1$
g_2 is constant, $g_2 \neq g_1$

III. SYSTEM DESCRIPTION

A parallel model following system is chosen:

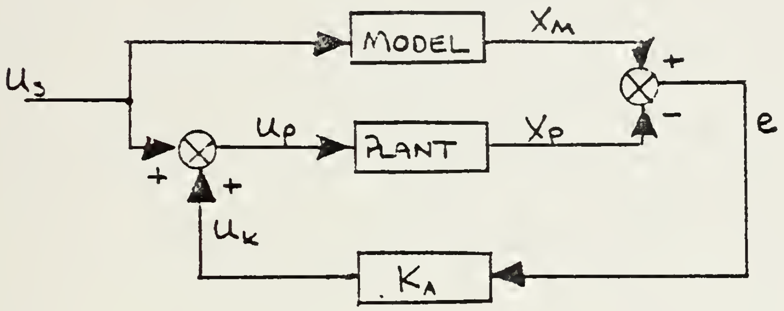

FIGURE 1. PARALLEL MODEL FOLLOWING CONTROL SYSTEM

where e is the magnitude of the system error:

$e = |x_m - x_p|$

K_a is the adaptive gain applied to e to generate u_K

u_K is the (additive) modification to the system control, u_s

In this system, the steady-state error (e_{ss}) can never be zero unless the plant and model have identical responses to the same input, since

$e_{ss} = 0$ implies that $u_K = 0$ implies that $u_p = u_s$

and

$e_{ss} = 0$ implies that $x_{mss} = x_{pss}$

Since this condition serves no useful purpose unless the parameters of the plant or model are adjustable, we introduce the idea of a constant (non-zero) error.

IV. CONSTANT ERROR MODEL FOLLOWING CONTROL

A. CONSTANT ERROR

Without changing the system, the error (e) is specified to be fixed at some arbitrary value (ε):

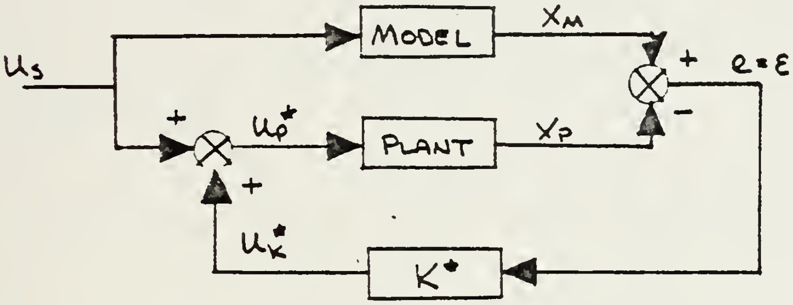

FIGURE 2. CONSTANT ERROR MODEL FOLLOWING CONTROL SYSTEM

We wish to determine the gain, K^*, which will insure that this condition, once established, will be maintained as shown in Figure 3.

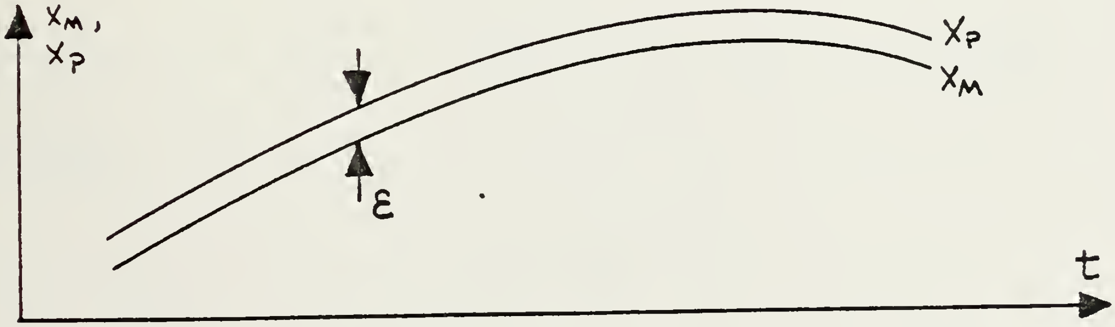

FIGURE 3. PLANT AND MODEL STATE TRAJECTORIES

The state trajectory of the plant follows that of the model with constant difference ε.

We have defined

$$e \triangleq |x_m - x_p| \qquad (4)$$

so that $\varepsilon \triangleq |x_m - x_p{}^*| = \text{constant}$,

where $x_p{}^*$ is the plant output for $e = \varepsilon$. Similarly, denote the value of K which maintains the error constant at $e = \varepsilon$ as K^* and the resulting control as u^* .

We now have

$$\varepsilon = |x_m - x_p{}^*| = (x_m - x_p{}^*) \operatorname{sgn} (x_m - x_p{}^*)$$

so

$$x_p{}^* = \frac{x_m \operatorname{sgn} (x_m - x_p{}^*) - \varepsilon}{\operatorname{sgn} (x_m - x_p{}^*)}$$

or

$$x_p{}^* = x_m - \varepsilon \operatorname{sgn} (x_m - x_p{}^*) \qquad (5)$$

and

$$\varepsilon = 0 = (x_m - x_p{}^*) \operatorname{sgn} (x_m - x_p{}^*)$$

($\frac{d}{dt} [\operatorname{sgn} (x_m - x_p{}^*)] = 0$ since $(x_m = x_p{}^*)$ is assumed constant)

so

$$\dot{x}_p{}^* = \dot{x}_m$$

The result is that, if the plant state is exactly $\pm \varepsilon$ from the model state, we may express $\dot{x}_p^*$ and x_p^* as functions of $\dot{x}_m$, x_m and $\pm \varepsilon$. The values of $\dot{x}_m$, x_m and ε are known for any time and system control because they have been specified.

Now note from the system description (Figure 2) that

$$u_p^* = u_s + u_K^*$$

or

$$u_K^* = u_p^* - u_s \tag{7}$$

and

$$u_K^* = K_a^* \varepsilon \tag{8}$$

Equations (1) and (2) may be solved for u_p^* and u_s :[1]

$$u_p^* = \frac{\dot{x}_p - (f_1 (x_p^* , t)}{g_1} \tag{9}$$

$$u_s = \frac{\dot{x}_m - f_2 (x_m , t)}{g_2} \tag{10}$$

By substituting equations (5) and (6) into equation (9):

$$u_p^* = \frac{\dot{x}_m - f_1 [(x_m - \varepsilon \operatorname{sgn} (x_m - x_p^*)) , t]}{g_1} \tag{11}$$

---

[1] If U_p and u_s are not additive controls as assume and if the state equations may be solved for u_p and u_s, the results which follow are still valid.

Here we define

$$f_1^* (x_m , \varepsilon , t) \triangleq f_1 [(x_m - \varepsilon \operatorname{sgn} (x_m - x_p^*)) , t] \tag{12}$$

$$= f_1 (x_p^* , t)$$

so
$$u_p^* = \frac{\dot{x}_p - f_1^* (x_m , \varepsilon , t)}{g_1} \tag{13}$$

From equations (7), (8), (10) and (13):

$$K^* \varepsilon = u^* = u^*_p - u_s$$

$$K^* \varepsilon = \frac{\dot{x}_m - f_1^* (x_m , \varepsilon , t)}{g_1} = \frac{\dot{x}_m - f_2 (x_m , t)}{g_2} \tag{14}$$

$$K^* = \frac{\dot{x}_m - f_1^* (x_m , \varepsilon , t)}{\varepsilon g_1} - \frac{\dot{x}_m - f_2 (x_m , t)}{\varepsilon g_2} \tag{15}$$

Equation (15) gives us the gain which will insure that, if $x_p = x_m \pm \varepsilon$ initially (Figure 4), the proper control will be applied to maintain it there if the system is undisturbed. It is expressed solely in terms of the model state and state rate, the specified error, and the functional relationships which define the plant and model responses.

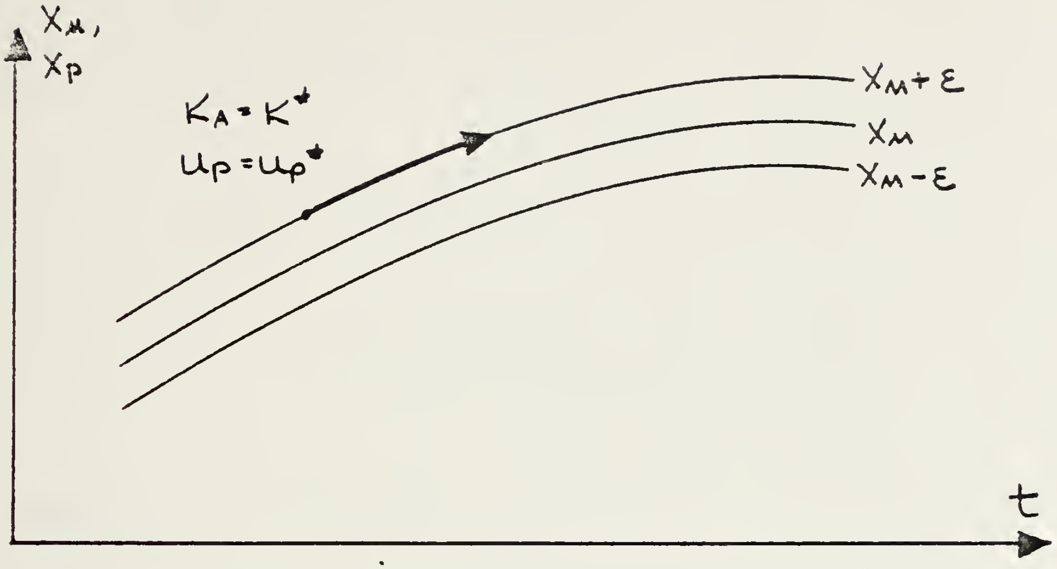

FIGURE 4. PLANT AND MODEL STATE TRAJECTORIES

The use of K^* gain is illustrated in the following example:

Example 1: Linear, first order, time invariant model and plant.

$$\left.\begin{aligned} \dot{x}_p &= a\,x_p + u_p \\ \dot{x}_m &= b\,x_m + u_s \end{aligned}\right\} \quad a \neq b$$

we have $$\dot{x}_p^* = a\,x_p^* + u_p^*$$

then
$$u_p{}^* = \dot{x}_p{}^* - a\ x_p{}^*$$

$$u_s = \dot{x}_m - b\ x_m$$

$$K_a{}^* = \frac{u_p{}^* - u_s}{\varepsilon}$$

$$= \frac{(\dot{x}_p - a\ x_p{}^*) - (\dot{x}_m - b\ x_m)}{\varepsilon}$$

From equations (5) and (6):

$$K_a{}^* = \frac{\dot{x}_m - a\ [x_m - \varepsilon\ \mathrm{sgn}\ (x_m - x_p{}^*)] - \dot{x}_m + b\ x_m}{\varepsilon}$$

or
$$K_a{}^* = \frac{(b - a)\ x_m + a\ \varepsilon\ \mathrm{sgn}\ (x_m - x_p{}^*)}{\varepsilon} \qquad (16)$$

The system is as shown in Figure 5.

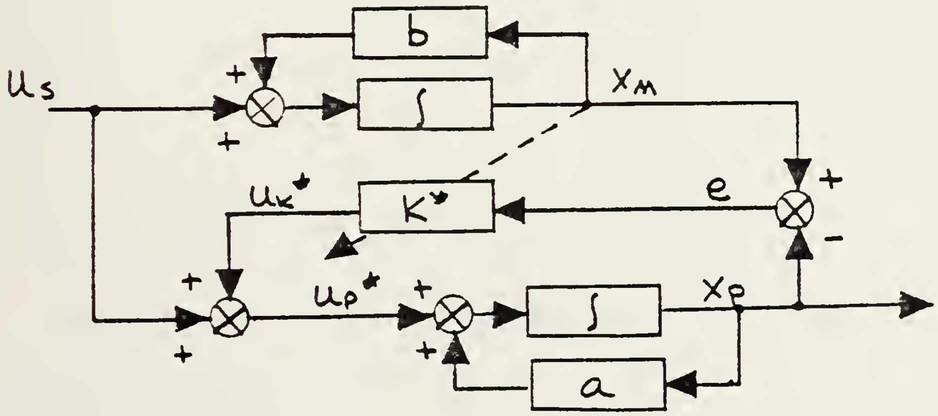

FIGURE 5. CONTROL SCHEMATIC FOR EXAMPLE 1

The system was simulated on the IBM 3033 using the Continuous System Modeling Program (CSMP). The following values were used:

$a = +\ 0.5$ (unstable impulse response)

$b = -\ 1.0$

$\varepsilon = 0.05$[2]

$x_m(0) = 0$

$x_p(0) = 0.1$

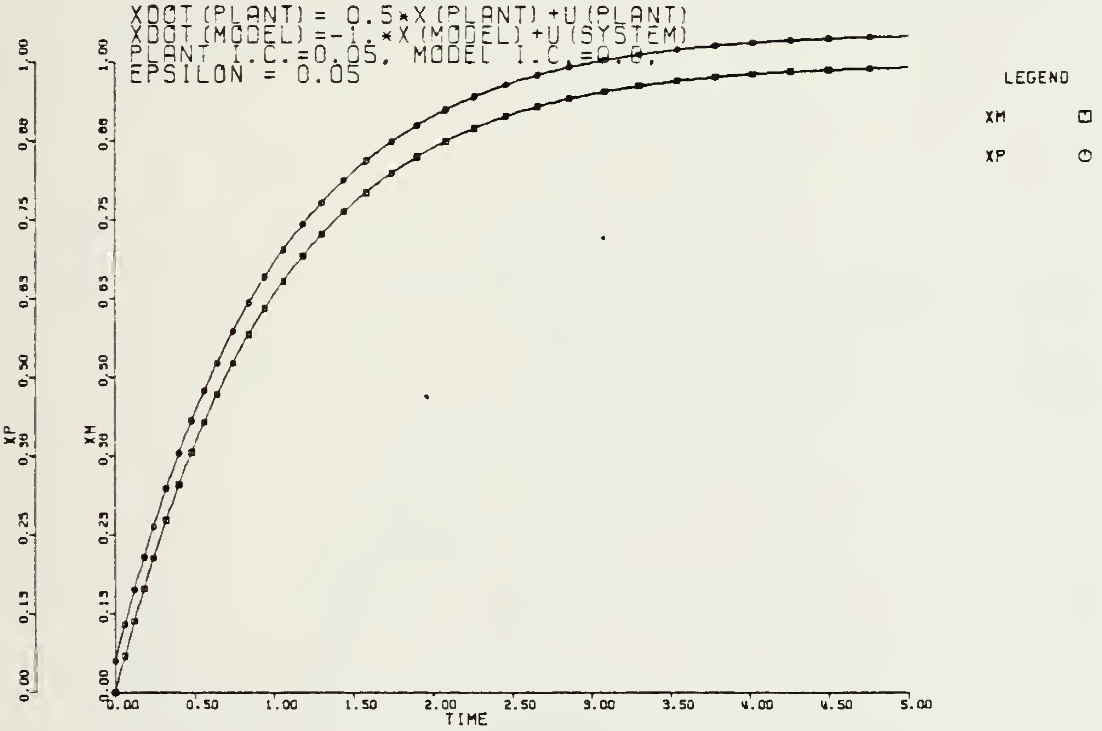

FIGURE 5a. SIMULATION RESULTS FOR EXAMPLE 1

---

[2] This value of ε was chosen so that the error following could be shown graphically. The system performs similarly for any value of $\varepsilon > 0$.

The input (u_s) was a unit step at $t = 0$. A time history of the system response for the first five seconds is shown in Figure 5a.

B. STABILITY OF THE SYSTEM USING K*e CONTROL

The question to be answered is: Will the gain K^* satisfactorily control the plant if $e \neq \varepsilon$?

We require that, in response to an error such that $e \neq \varepsilon$, the system tend toward $e = \varepsilon$. That is, the state of the plant, x_p, should tend toward $x_m \pm \varepsilon$ (static stability).

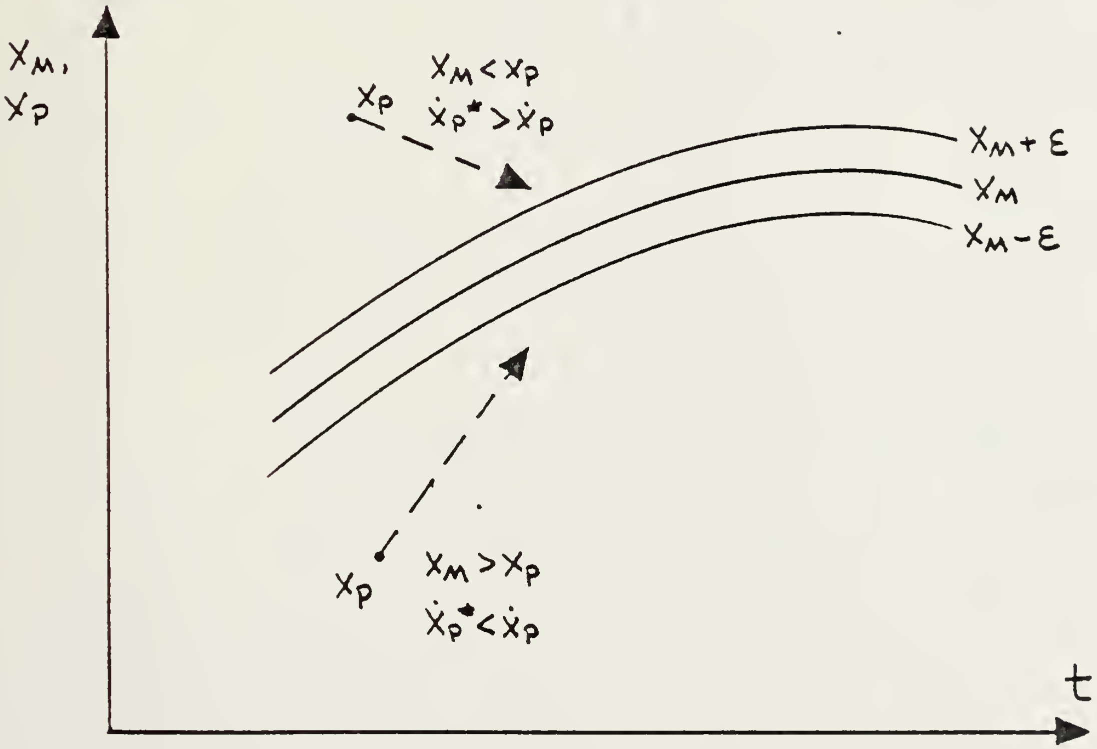

FIGURE 6. PLANT TRAJECTORIES FOR DECREASING THE ERROR

If $e > \varepsilon$, we require $\dot{e} < 0$:

$$\dot{e} = (\dot{x}_m - \dot{x}_p) \operatorname{sgn} (x_m - x_p) < 0 \quad \text{if} \quad e > \varepsilon \tag{17}$$

(Again, the Signum function has no derivative since the plant state trajectory does not cross that of the model.)

We have $\dot{x}_p{}^* = \dot{x}_m$ (6) so,

$$\dot{e} = (\dot{x}_p{}^* - \dot{x}_p) \operatorname{sgn} (x_m - x_p) < 0 \quad \text{if} \quad e > \varepsilon \tag{18}$$

The two cases are $x_m > x_p$ and $x_m < x_p$:

$$\left.\begin{array}{l} x_m > x_p \implies \dot{x}_p{}^* < \dot{x}_p \\ \\ x_m < x_p \implies \dot{x}_p{}^* > \dot{x}_p \end{array}\right\} \text{for} \quad \dot{e} < \varepsilon \tag{19}$$

These two cases are shown in Figure 6.

From equation (1):

$$\left.\begin{array}{l} \dot{x}_p = f_1 (x_p , t) + g_1 K^* e \\ \\ \dot{x}_p{}^* = f_1 (x_p{}^* , t) + g_1 K^* \varepsilon \end{array}\right\} \tag{20}$$

In determining whether $\dot{x}_p{}^* < \dot{x}_p$ or $\dot{x}_p{}^* > \dot{x}_p$, we know $e > \varepsilon$, and we can examine $f_1 (x_p , t)$ to see if

it increases or decreases with x_p. But the sign and magnitude of K^* will be determined by the model being followed (equation (15)).

From equation (20) we have

$$\dot{x}_p = \dot{x}_p^* = [f_1 (x_p , t) - f_1 (x_p^* , t)] + g_1 K^* (e - \varepsilon) \quad (21)$$

From equation (19), with $e > \varepsilon$,

First Case: $\underline{x_m > x_p} \Rightarrow \dot{x}_p^* < x_p \Rightarrow$

$$[f_1 (x_p , t) - f_1 (x_p^* , t)] + g_1 K^* (e - \varepsilon) > 0$$

or $$[f_1 (x_p , t) - f_1 (x_p^* , t)] > - g_1 K^* (e - \varepsilon) \quad (22a)$$

Likewise,

Second Case: $\underline{X_m < x_p} \Rightarrow$

$$[f_1 (x_p , t) - f_1 (x_p^* , t)] < - g_1 K^* (e - \varepsilon) \quad (22b)$$

For any given x_p, x_m, ε and g_1, all the quantities in equations (22a) and (22b) except K^* are determined. Since we wish our descriptions of the plant and the model to be arbitrary, we cannot assure that equations (22a) and (22b) will hold for all cases.

This is illustrated by Example 1, where

$$\dot{x}_p = a\, x_p + u_p$$

$$\dot{x}_m = b\, x_m + u_s$$

Using equations (22a) and (22b):

$x_m > x_p$ requires

$$a\, x_p - a\, x_p^* > - K^* (e - \varepsilon)$$

$$a\, (x_p - x_p^*) > - K^* (e - \varepsilon)$$

For $x_m > x_p$, $(x_p - x_p^*) = - (e - \varepsilon)$

since $(e - \varepsilon) > 0$, we require

$$- a > - K^* \quad \text{or} \quad a < K^*$$

Similarly, for $x_m < x_p$ we require

$$a < - K^*$$

Thus, if $a < 0$, we require $|a| < |K^*|$, or

$$|a| < \left| \frac{(b - a)\, x_m + a\, \varepsilon\, \operatorname{sgn}\, (x_m - x_p^*)}{\varepsilon} \right|$$

using equation (15). This places an unacceptable restriction on the choice of model and ε . If $a > 0$, one or the other of the inequalities is not satisfied.

Using the values given in Example 1, the value of K^* at $t = 1.5$ seconds was found (from the simulation) to be -12.2. Here, $a = 0.5$, a is not less than K^* and we can expect divergence if $x_m > x_p$.

A negative step disturbance of magnitude 0.3 was superimposed on x_p at $t = 1.5$ seconds in the simulation. The resulting divergence is shown in Figure 7.

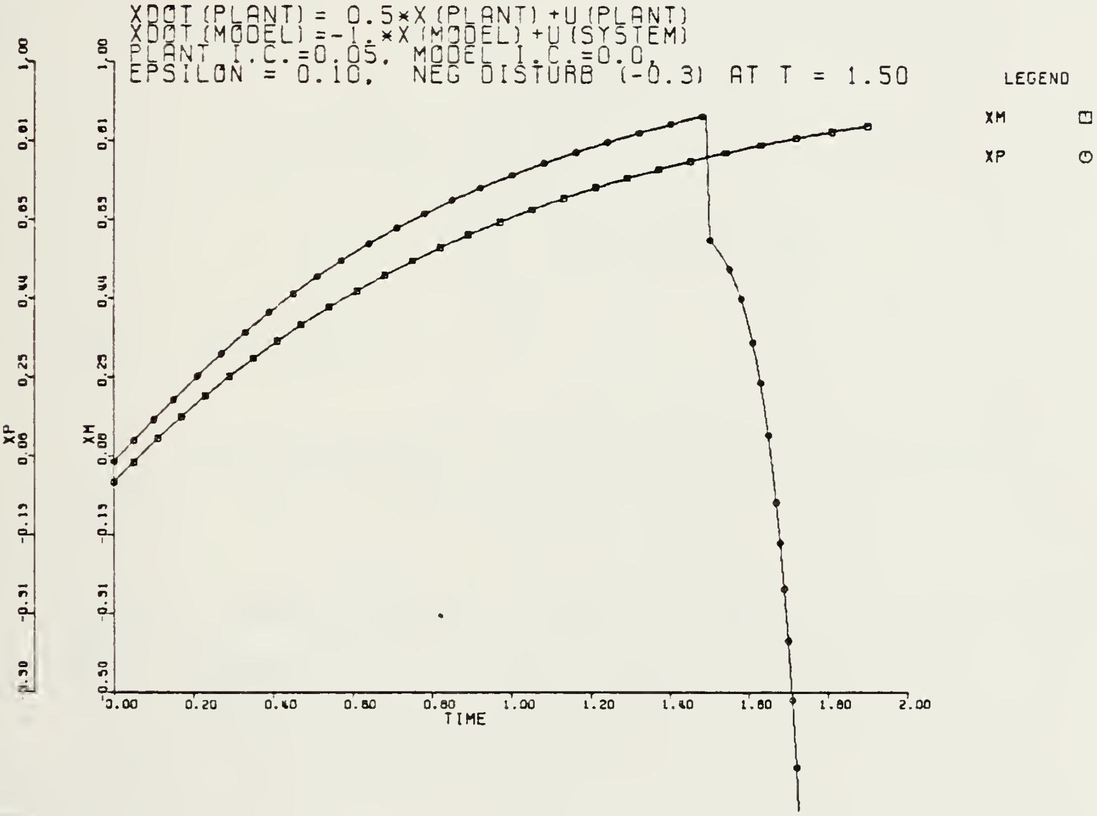

FIGURE 7. DIVERGENCE RESULTING FROM K^*e CONTROL

C. STABILIZING THE SYSTEM

Neutral static stability $(\dot{e} = 0)$ can be established by taking $\varepsilon = e$. That is, if an error (e) is present in the system, we take that error as our new ε. If the error changes, we change ε. Equation (15) becomes

$$K_N = \frac{\dot{x}_m - f_{1N}(x_m , e , t)}{e\, g_1} - \frac{\dot{x}_m - f_2(x_m , t)}{e\, g_2} \qquad (23)$$

where the subscript N denotes a neutrally stable system, and

$$f_{1N}(x_m , e , t) \triangleq f_1(x_{pN} , t)$$

$$x_{pN} = x_m - e \operatorname{sgn}(x_m - x_p)$$

Since $u_{KN} = K_N e$

$$u_{KN} = \frac{\dot{x}_m - f_{1N}(x_m , e , t)}{g_1} - \frac{\dot{x}_m - f_2(x_m , t)}{g_2} \qquad (24)$$

At this point we are no longer computing a gain, but directly synthesizing a control modification using $\dot{x}_m$, x_m, e and the descriptions of plant and model dynamics.

Equation (24) gives the control which, when added to the system control, will cause the plant to follow the model with constant error. If the error present in the

system should change, the plant will follow the model with the new error held constant. This is illustrated by way of Example 1. Equation (16) becomes

$$u_{KN} = (b - a)\, x_m + a\, e\, \text{sgn}\, (x_m - x_p) \qquad (25)$$

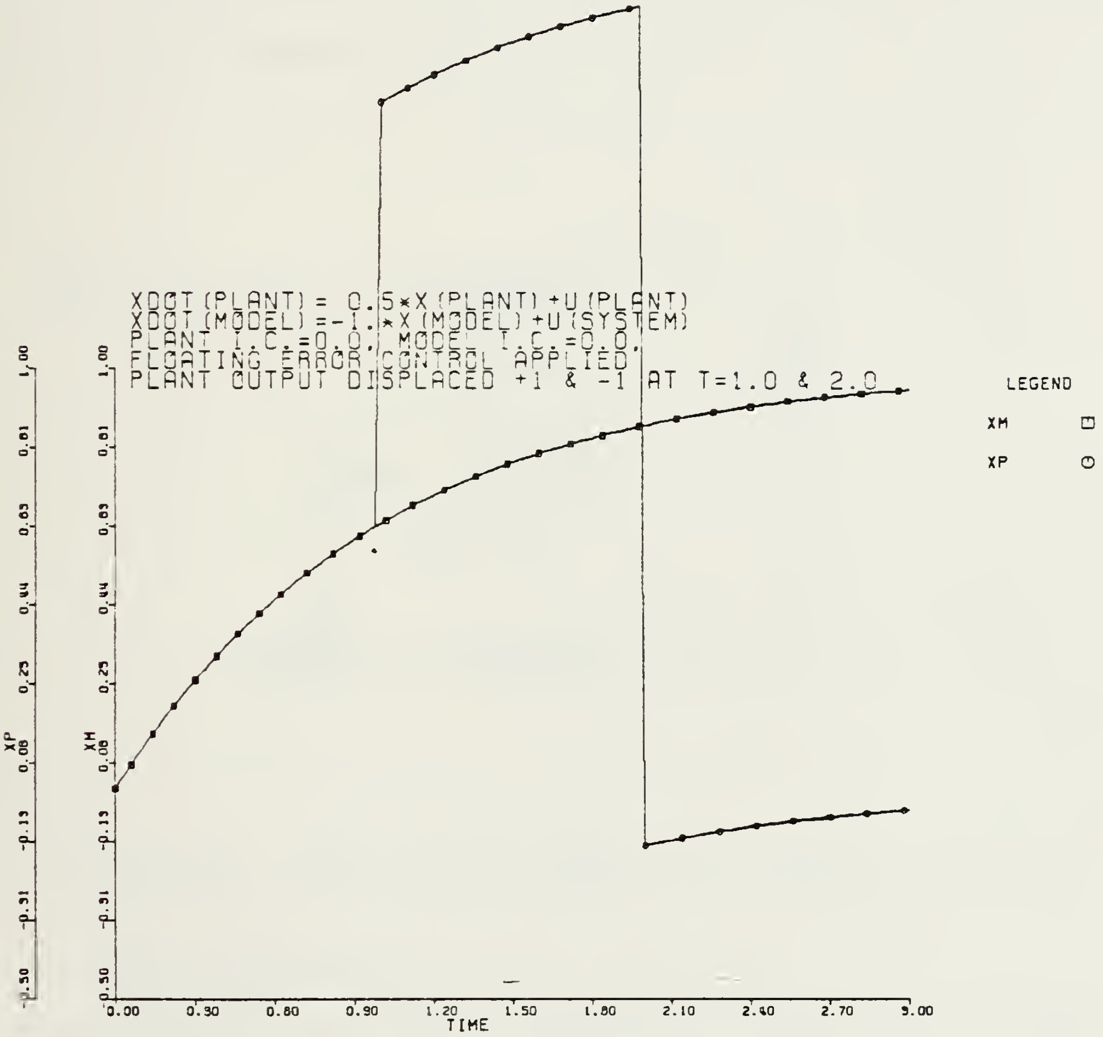

FIGURE 8. CONSTANT ERROR CONTROL SIMULATION

The simulation was run with x_p being disturbed by step inputs at $t = 1.0$ and $t = 2.0$ seconds as shown in Figure 8.

Note that the error is constant even with $e = 0$. This does not alter the discussion following Figure 1 regarding steady state zero error, because we are no longer calculating a gain as shown in Figure 1. The present form of the system is shown in Figure 9.

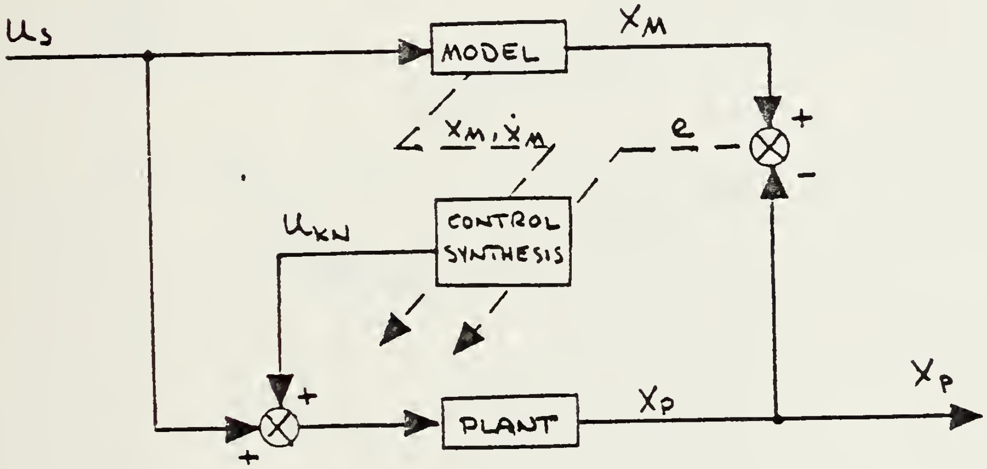

FIGURE 9. FORM OF THE SYSTEM WITH CONTROL SYNTHESIS

At this point, if

A. The physical pant is perfectly described by the plant equation, and

B. The plant and the model have the same initial conditions $(e(0) = 0)$, and

C. There are no external disturbances (or if the mean disturbance is zero), then the control defined by equation (24) will yield zero error (or zero mean error).

D. ERROR AND ERROR RATE CONTROL

If an error exists in the system, it is desired that it be reduced to zero in a controlled manner. Figure 10 shows a system with initial error. The trajectory of x_p with constant error (equation (24)) is shown, as well as a desired response to this condition (dashed line).

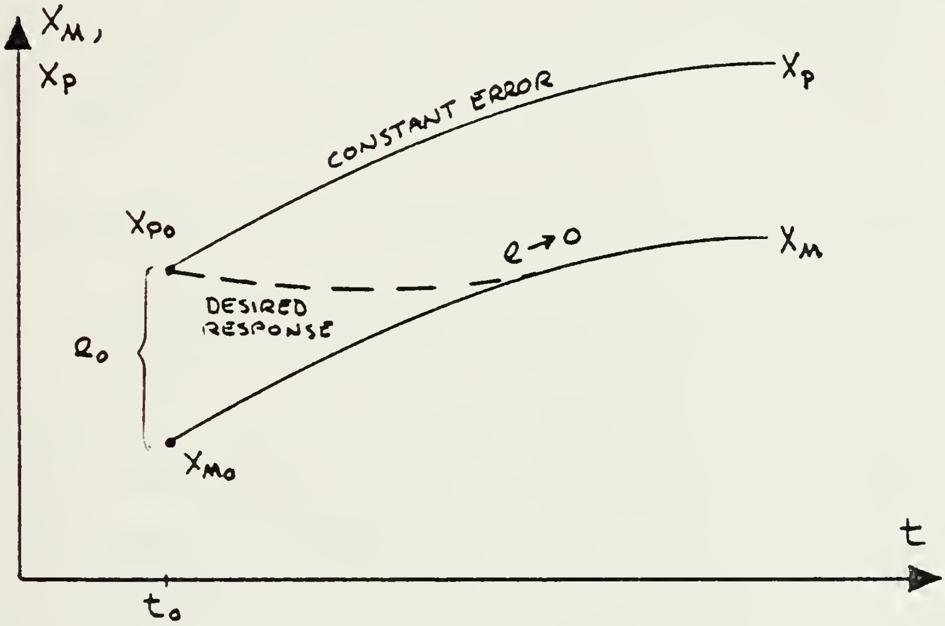

FIGURE 10. CONSTANT ERROR AND DESIRED RESPONSE TRAJECTORIES OF THE PLANT

The desired response may be obtained by observing that the plant may be made to follow any model with constant error. That is, if we can find another model (to be called

the control model) which, if followed with constant error, will cause the plant state to return to the model trajectory, the desired response is obtained.

The trajectory of the control model is shown in Figure 11.

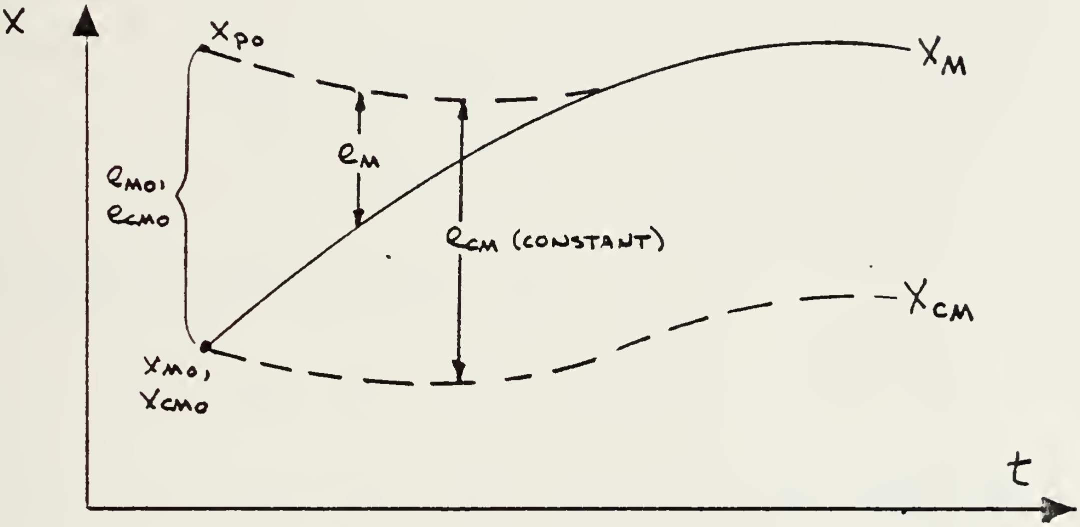

FIGURE 11. CONTROL MODEL TRAJECTORY

The notations e_m and e_{cm} represent the errors between the plant and the model, and between the plant and the control model, respectively. Note that e_{cm} is held constant, and that the trajectory of x_{cm} causes e_m to diminish.

From Figure 10 the nature of x_{cm} is seen: If e_m is 0 , then x_{cm} is the same as x_m (except, perhaps, for initial conditions). If e_m is non-zero, then $\dot{x}_{cm}$ differs from $\dot{x}_m$ by an amount proportional to e_m .

A control model which achieves the desired response may be written:

$$\dot{x}_{cm} = \dot{x}_m \, \Delta \, (e_m) \, \text{sgn} \, (x_m - x_p) \tag{26}$$

where $\Delta \, (e_m) \geq 0$ and

$\Delta \, (e_m) = 0$ if

$e_m = 0$

The Signum function assures that

$\dot{x}_{cm} < \dot{x}_m$ if $x_p > x_m$ and

$\dot{x}_{cm} > \dot{x}_m$ if $x_p < x_m$, as required.

From equation (2):

$$\dot{x}_{cm} = f_2 \, (x_m \, , \, t) + g_2 \, u_s + \Delta \, (e_m) \, \text{sgn} \, (x_m - x_p) \tag{27}$$

so

$$u_s = \frac{\dot{x}_{cm} - f_2 \, (x_m \, , \, t) - \Delta \, (e_m) \, \text{sgn} \, (x_m - x_p)}{g_2} \tag{28}$$

The additive control input necessary to follow the control model with constant error is then (from equation (24)):

$$u_K = \frac{\dot{x}_{cm} - f_{1N}(x_{cm}, e_{cm}, t)}{g_1} - \frac{\dot{x}_{cm} - f_2(x_m, t) - \Delta(e_m)\,\mathrm{sgn}(x_m - x_p)}{g_2} \tag{29}$$

In equation (29) $f_{1N}(x_{cm}, e_{cm}, t)$ is defined as the function evaluated at

$$x_p = x_{cm} - e_{cm}\,\mathrm{sgn}(x_{cm} - x_p)$$

This expression may be shown to be independent of x_{cm} by observing the six possible relationships between x_p, x_m and x_{cm} and writing the expression for e_{cm} for each:

$$\begin{array}{ll}
x_p > x_m > x_{cm} & e_{cm} = x_m - x_{cm} + e_m \\
x_p > x_{cm} > x_m & e_{cm} = x_m - x_{cm} + e_m \\
x_m > x_p > x_{cm} & e_{cm} = x_m - e_{cm} - e_m \\
x_m > x_{cm} > x_p & e_{cm} = - x_m + x_{cm} + e_m \\
x_{cm} > x_p > x_m & e_{cm} = - x_m + x_{cm} - e_m \\
x_{cm} > x_m > x_p & e_{cm} = - x_m + x_{cm} + e_m
\end{array}$$

From which, for all cases

$$e_{cm} = x_{cm} \operatorname{sgn}(x_{cm} - x_p) - x_m \operatorname{sgn}(x_{cm} - x_p)$$

$$+ e_m \operatorname{sgn}(x_m - x_p) \operatorname{sgn}(x_{cm} - x_p)$$

then

$$e_{cm} \operatorname{sgn}(x_{cm} - x_p) = x_{cm} - x_m + e_m \operatorname{sgn}(x_m - x_p)$$

so $$x_{cm} - e_{cm} \operatorname{sgn}(x_{cm} - x_p) = x_m - e_m \operatorname{sgn}(x_m - x_p)$$

The expression for u_K is then

$$u_K = \frac{\dot{x}_{cm} - f_{1N}(x_m, e_m, t)}{g_1} - \frac{\dot{x}_{cm} - f_2(x_m, t) - \Delta(e_m) \operatorname{sgn}(x_m - x_p)}{g_2} \qquad (30)$$

where $f_{1N}(x_m, e_m, t)$ is the function f_1 evaluated at $x_p = x_m - e_m \operatorname{sgn}(x_m - x_p)$.

In equation (30), if $g_1 = g_2$ then u_K does not depend on $\dot{x}_{cm}$.

In any case, it is simpler at this point to replace the last term in equation (30) by u_s and calculate directly $u_p = u_K + u_s$, or

$$u_p = \frac{\dot{x}_{cm} \; f_{1N} \; (x_m \; , \; e_m \; , \; t)}{g_1} \tag{31}$$

The selection of the function $\Delta \, (e_m)$ appears to be arbitrary with unconstrained control. The plant following the control model in exactly the desired manner in each simulation run. The form of $\Delta \, (e_m)$ chosen for most simulations was $\Delta \, (e_m) = [\exp \, (w \, . \, e_m) - 1]$, where w was selected to vary the speed of the response.

The application of this form of error control is illustrated by continuing Example 1:

$$\left.\begin{aligned} \dot{x}_p &= a \; x_p = u_p \\ \\ \dot{x}_m &= b \; x_m + u_s \end{aligned}\right\} \; a \neq b$$

From equation (26), $\dot{x}_{cm} = b \; c_m + u_s + \Delta \, (e_m) \; \mathrm{sgn} \; (x_m - x_p)$

From equation (30), $u_K = \dfrac{\dot{x}_{cm} \; a \; (x_m - e_m \; \mathrm{sgn} \; (x_m - x_p))}{1}$

$$- \frac{\dot{x}_{cm} - b \; x_m - \Delta \, (e_m) \; \mathrm{sgn} \; (x_m - x_p)}{1}$$

The expression for u_K simplifies to

$$u_K = b\, x_m + \Delta\, (e_m)\ \mathrm{sgn}\ (x_m - x_p)$$
$$- a\, (x_m - e_m\ \mathrm{sgn}\ (x_m - x_p))$$

selecting $\Delta\, (e_m) = [\exp\, (3\, e_m) - 1]$.

The problem was simulated with the values $a = +\,0.5$, and $b = -\,1.0$ as before. The plant, model, and control model had the same initial conditions (zero). A step disturbance was imposed on the plant at $t = 0.5$ second. The response is shown in Figure 12:

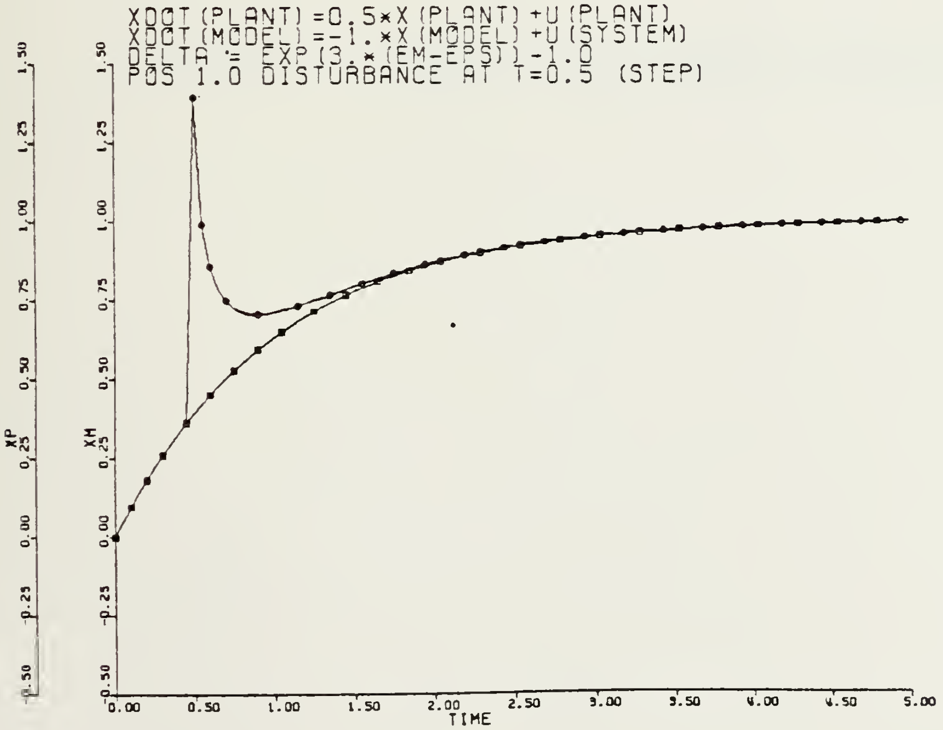

FIGURE 12. SIMULATION USING CONSTANT ERROR MODEL FOLLOWING CONTROL

The system at this point is most simply described from equation (31) as a feedforward control synthesizer with error feedback, as shown in Figure 13:

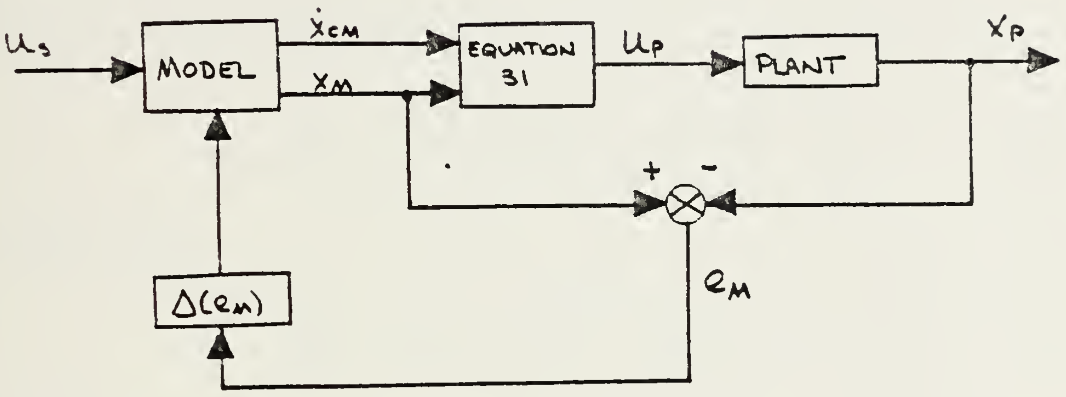

FIGURE 13. CONTROL SYSTEM CONFIGURATION WITH CONSTANT ERROR MODEL FOLLOWING CONTROL

V. SUMMARY

The method of constant error model following control is summarized as follows:

Given the plant described by

$$\dot{x}_p = f_1 (x_p , t) + g_1 u_p \tag{1}$$

and the model

$$\dot{x}_m = f_2 (x_m , t) + g_2 u_s \tag{2}$$

Define the control model

$$\dot{x}_{cm} = \dot{x}_m + \Delta (e_m) \operatorname{sgn} (x_m - x_p) \cdot \tag{26}$$

Select $\Delta (e_m)$ such that

$$\Delta (e_m) \geq 0$$

$$\Delta (0) = 0$$

$$\Delta (e_{m1}) > \Delta (e_{m2}) \quad \text{if} \quad e_{m1} > e_{m2}$$

Solve equation (1) for u_p :

$$u_p = \frac{\dot{x}_p - f_1 (x_p , t)}{g_1} \tag{32}$$

In equation (32) substitute

$$\dot{x}_p = \dot{x}_{cm}$$

$$f_1 (x_p , t) = f_{1N} (x_m , e_m , t)$$

where

$$f_{1N} (x_m , e_m , t) = f_1 (x_p , t) \quad \text{with}$$

$$x_p = x_N - e_m \operatorname{sgn} (x_m - x_p)$$

The result

$$u_p = \frac{\dot{x}_{cm} - f_{1N} (x_m , e_m , t)}{g_1} \tag{31}$$

Is the plant input which makes the plant states equal to those of the model, and restores this condition in the presence of error.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. It is concluded that constant error model following control achieves the desired goal of error control for the systems described in this paper. It offers the advantage of perceptual simplicity in that the designer can visualize exactly the effect of his control method on the system response. It affords flexibility in that the choice of the restoring function $\Delta\ (e_m)$ is arbitrary within the few constraints mentioned.

B. The following areas of future research are suggested:

1. Development of the theory with application to higher order systems with non-additive controls.
2. Investigation of the response of systems in which the physical plant is not accurately described by the plant equations.
3. Application of the theory to practical problems.

APPENDIX A: EXAMPLE OF APPLICATION OF THE METHOD TO A FIRST ORDER NONLINEAR SYSTEM WITH TIME VARYING COEFFICIENTS

1. Assume the Plant is given by

$$\dot{x}_p = tx_p^2 + 2up \tag{A1}$$

and the model by

$$\dot{x}_m = -(t\,x_m)^{1/2} + u_s \tag{A2}$$

define

$$\dot{x}_{cm} = \dot{x}_m + \Delta\,(e_m)\ \mathrm{sgn}\,(x_m - x_p) \tag{A3}$$

then

$$u_p = \frac{\dot{x}_p - tx_p^2}{2} \tag{A4}$$

into (A5) substitute

$$\dot{x}_p = \dot{x}_{cm} \tag{A5}$$

$$x_p = x_m - e_m\ \mathrm{sgn}\,(x_m - x_p) \tag{A6}$$

then

$$u_p = \frac{-(tx_m)^{1/2} + u_s + \Delta(e_m) \operatorname{sgn}(x_m - x_p) - t(x_m - e_m \operatorname{sgn}(x_m - x_p)}{2} \tag{A7}$$

2. The system was simulated using the Continuous System Modeling Program (CSMP) on the IBM-3033. Time histories of the response following a disturbance at $t = 0.5$ second are shown in Figures A1 through A8. The system control input is a unit step at $t = 0$ in all cases. Shown also are the plant controls generated in each case. The four cases were the same except for the choice of $\Delta(e_m)$:

A. $\Delta(e_m) = 5e_m$ (Figures A1 and A2)
$\Delta(e_m) = \exp(e_m) - 1$ (Figures A3 and A4)
$\Delta(e_m) = \exp(2e_m) - 1$ (Figures A5 and A6)
$\Delta(e_m) = \exp(3e_m) - 1$ (Figures A7 and A8)

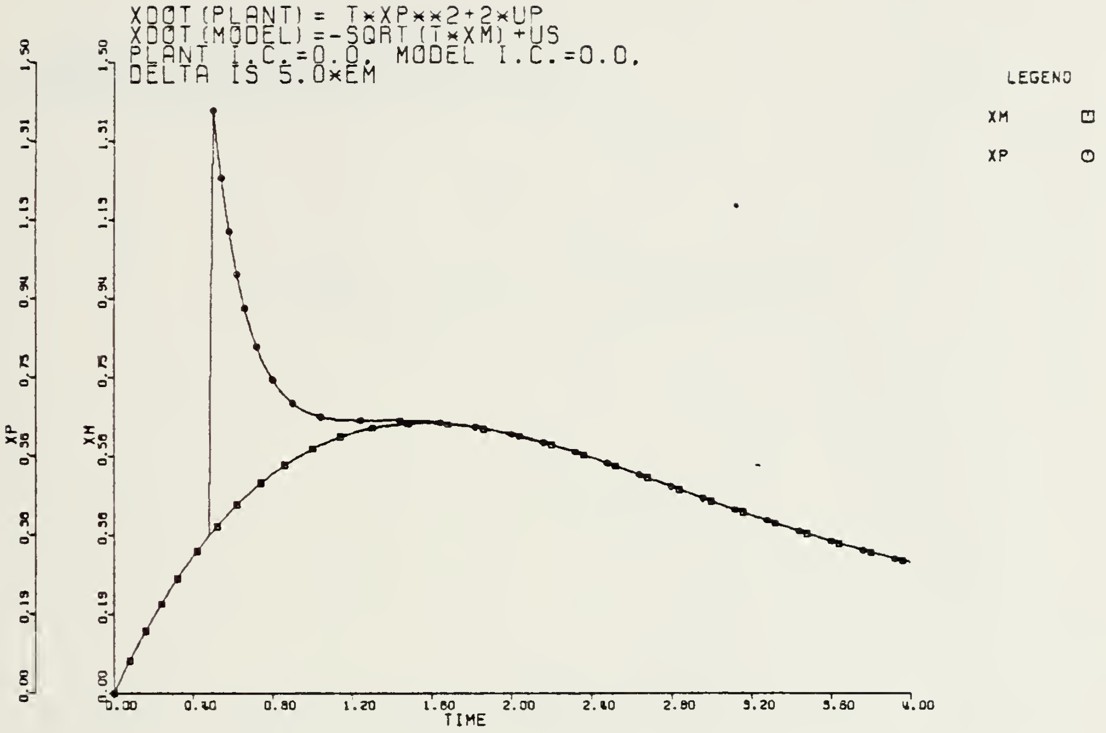

FIGURE A1. MODEL AND PLANT RESPONSES,

$\Delta(e_m) = 5e_m$.

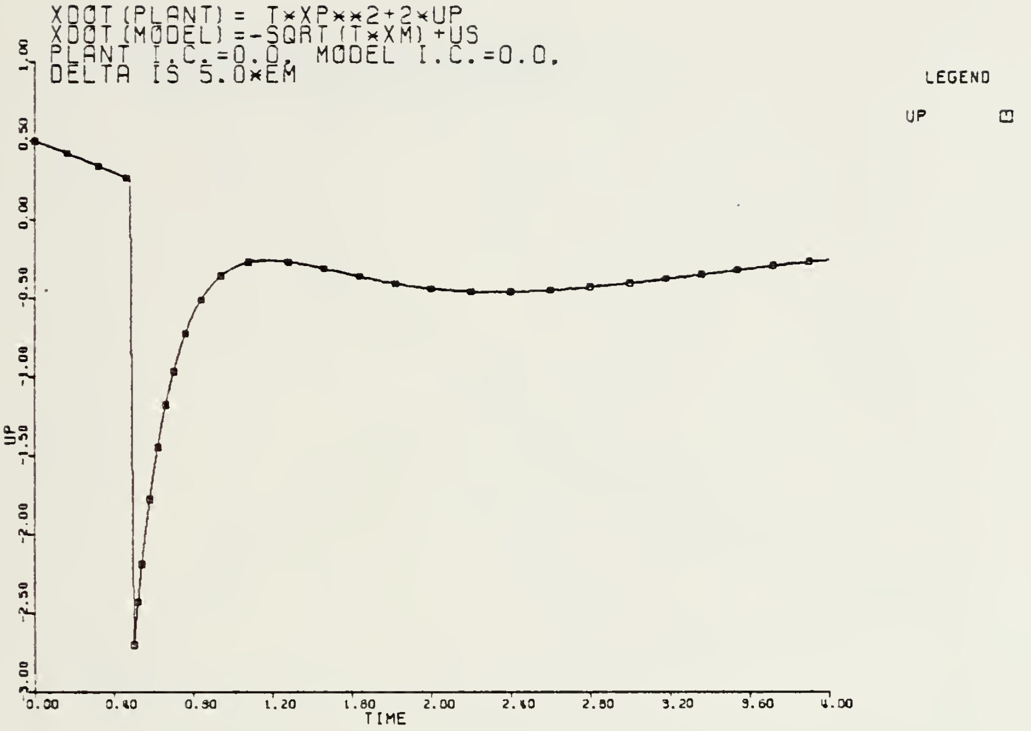

FIGURE A2: PLANT CONTROL, $\Delta(e_m) = 5e_m$.

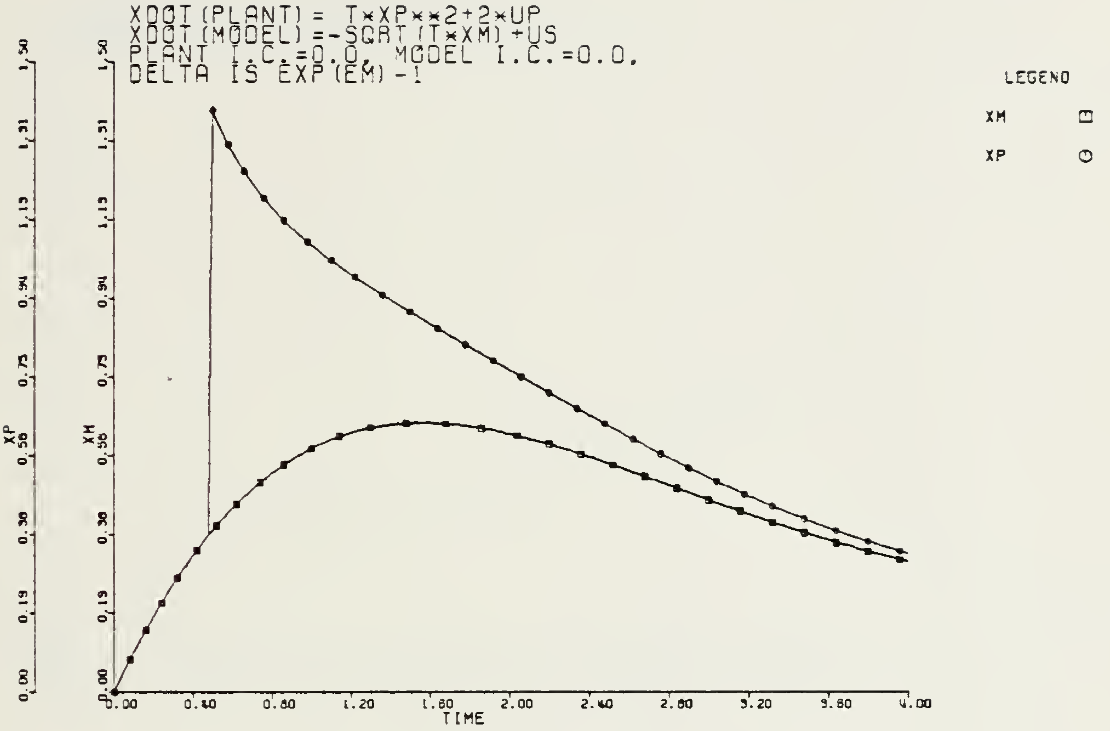

FIGURE A3: MODEL AND PLANT RESPONSES, $\Delta(e_m) = \exp(e_m) - 1$.

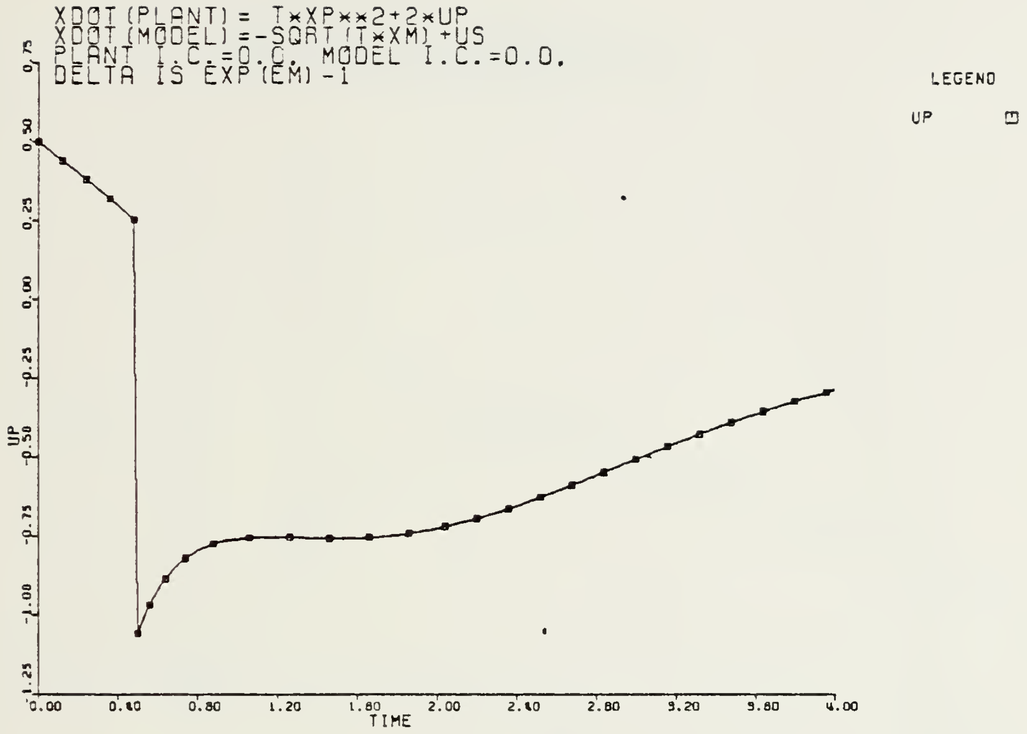

FIGURE A4: PLANT CONTROL, $\Delta(e_m) = \exp(e_m) - 1$.

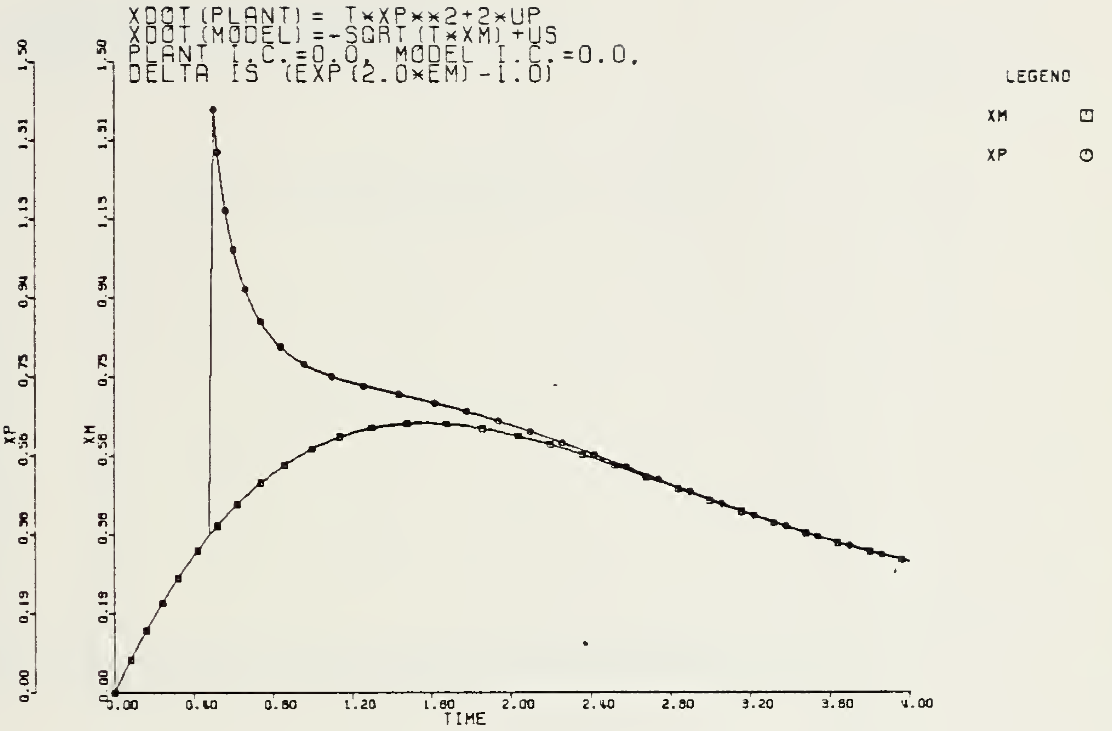

FIGURE A5: MODEL AND PLANT RESPONSES, $\Delta(e_m) = \exp(2\ e_m) - 1$.

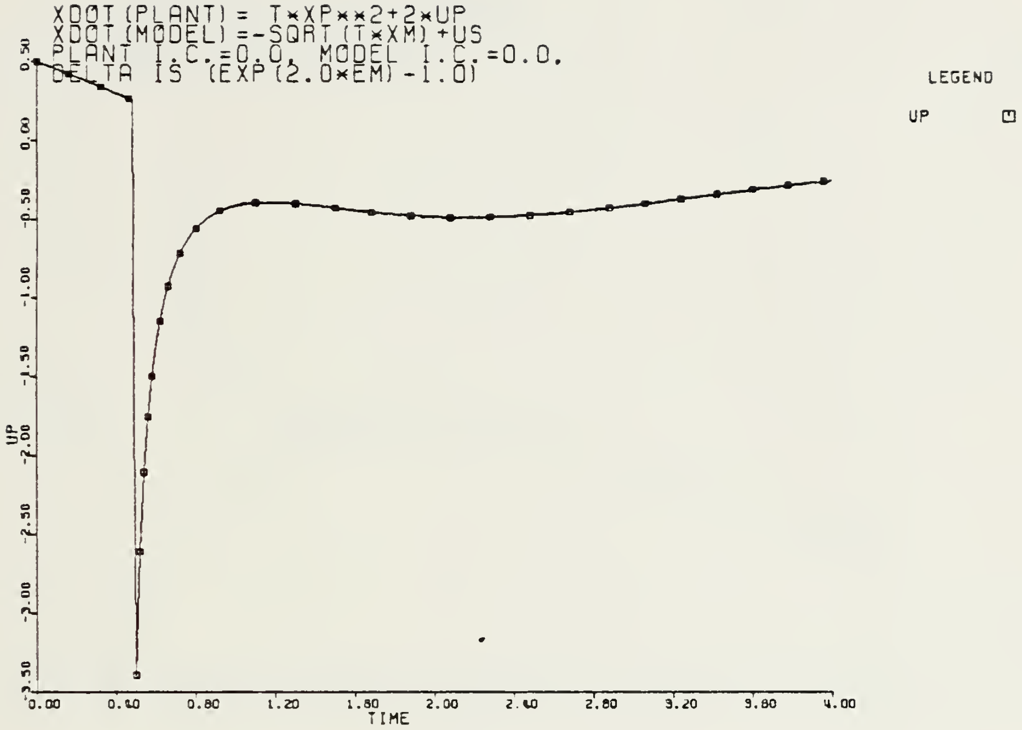

FIGURE A6: PLANT CONTROL, $\Delta(e_m) = \exp(2\ e_m) - 1$.

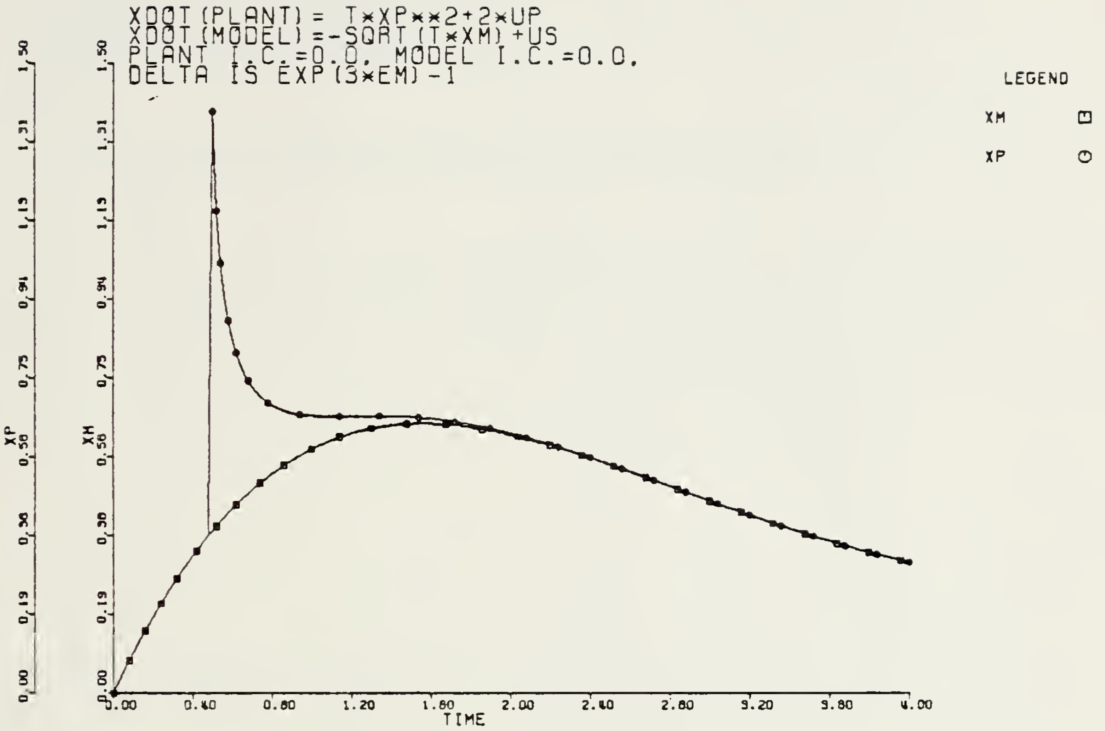

FIGURE A7: MODEL AND PLANT RESPONSES, $\Delta(e_m) = \exp(3 e_m) - 1$.

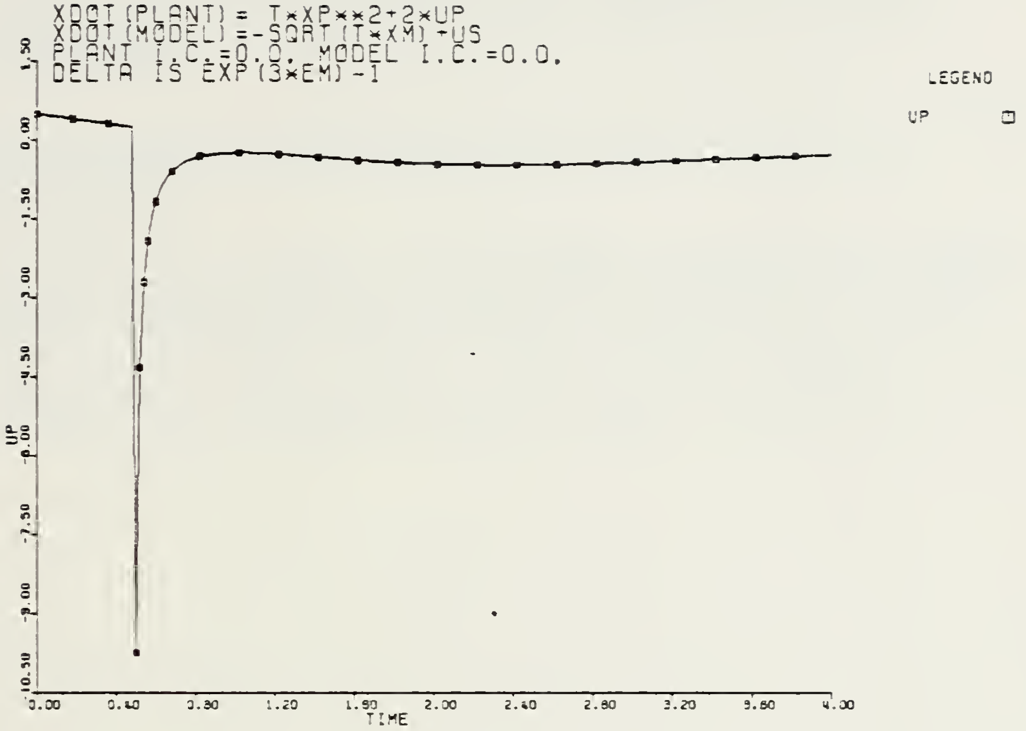

FIGURE A8: PLANT CONTROL, $\Delta(e_m) = \exp(3\,e_m) - 1$.

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